SHAMBHUNATH INSTITUTE OF ENGINEERING AND TECHNOLOGY

Subject Code : RME-603 **Subject :MACHINE DESIGN II**

B.Tech. 6TH SEM

FIRST SESSIONAL EXAMINATION, EVEN SEMESTER, (2019-2020)
MECHANICAL ENGINEERING
SOLUTION

SECTION – A

Answer (a) The **contact ratio** is **defined** as the **ratio** of the length of arc of **contact** (from lowest point to the highest point at **contact** exit) to the circular pitch. In other words, **contact ratio** is the average number of teeth in mesh during a **contact** cycle.

Answer (b) The basic rack is a segment of a gear of infinite radius belonging to a system of conjugate gearing in which the tooth profiles are such that any one gear will mate with any other. The pitch line of the basic rack is a straight line. The basic rack of involute gears has straight-sided profiles.

Answer (c) **Pressure angle** in relation to **gear** teeth, also known as the **angle** of obliquity, is the **angle** between the tooth face and the **gear** wheel tangent. It is more precisely the **angle** at a pitch point between the line of **pressure** (which is normal to the tooth surface) and the plane tangent to the pitch surface.

Answer (d) The profile of gear teeth are designed such that they will produce constant angular velocity ratio during meshing, and this is called conjugate action.The profile of the meshing teeth are said to be conjugate profile.

Answer (e) The arc of approach is the arc of pitch circle through which a tooth moves from its beginning of contact until the point of contact arrives at the pitch point.

SECTION – B

Answer (2a)

10.3 LAW OF GEARING

The law of gearing states the condition which must be fulfilled by the gear tooth profiles to maintain a constant angular velocity ratio between two gears. Figure 10.17 shows two bodies 1 and 2 representing a portion of the two gears in mesh.

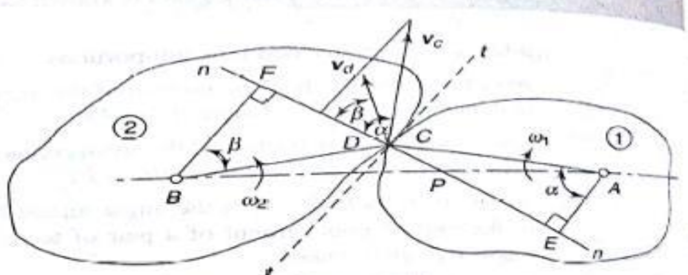


Fig. 10.17

A point C on the tooth profile of the gear 1 is in contact with a point D on the tooth profile of the gear 2. The two curves in contact at points C or D must have a common normal at the point. Let it be $n - n$.

Let ω_1 = instantaneous angular velocity of the gear 1 (clockwise)

ω_2 = instantaneous angular velocity of the gear 2 (counter-clockwise)

v_c = linear velocity of C

v_d = linear velocity of D

Then $v_c = \omega_1 . AC$ in a direction perpendicular to AC or at an angle α to $n - n$.

$v_c = \omega_2 . BD$ in a direction perpendicular to BD or at an angle β to $n - n$.

Now, if the curved surfaces of the teeth of two gears are to remain in contact, one surface may slide relative to the other along the common tangent $t - t$. The relative motion between the surfaces along the common normal $n - n$ must be zero to avoid the separation, or the penetration of the two teeth into each other.

Component of v_c along $n - n = v_c \cos\alpha$

Component of v_d along $n - n = v_d \cos\beta$

Relative motion along $n - n = v_c \cos\alpha - v_d \cos\beta$

Draw perpendiculars AE and BF on $n - n$ from points A and B respectively. Then $\angle CAE = \alpha$ and $\angle DBF = \beta$. For proper contact,

$$v_c \cos\alpha - v_d \cos\beta = 0$$

or $\omega_1 AC \cos\alpha - \omega_2 BD \cos\beta = 0$

or $\omega_1 AC \dfrac{AE}{AC} - \omega_2 BD \dfrac{BF}{BD} = 0$

or $\omega_1 AE - \omega_2 BF = 0$

or $$\frac{\omega_1}{\omega_2} = \frac{BF}{AE}$$

$$= \frac{BP}{AP} \qquad [\because \Delta AEP \text{ and } BEP \text{ are similar}]$$

Thus, it is seen that the centre line AB is divided at P by the common normal in the inverse ratio of the angular velocities of the two gears. If it is desired that the angular velocities of two gears remain constant, the common normal at the point of contact of the two teeth should always pass through a fixed point P which divides the line of centres in the inverse ratio of angular velocities of two gears.

As seen earlier, P is also the point of contact of two pitch circles which divides the line of centres in the inverse ratio of the angular velocities of the two circles and is the pitch point.

Thus, for constant angular velocity ratio of the two gears, the common normal at the point of contact of the two mating teeth must pass through the pitch point.

Also, as the Δs AEP and BFP are similar,

$$\frac{BP}{AP} = \frac{FP}{EP}$$

or $$\frac{\omega_1}{\omega_2} = \frac{FP}{EP} \quad \text{or} \quad \omega_1 EP = \omega_2 FP \tag{10.4}$$

Answer (2b)

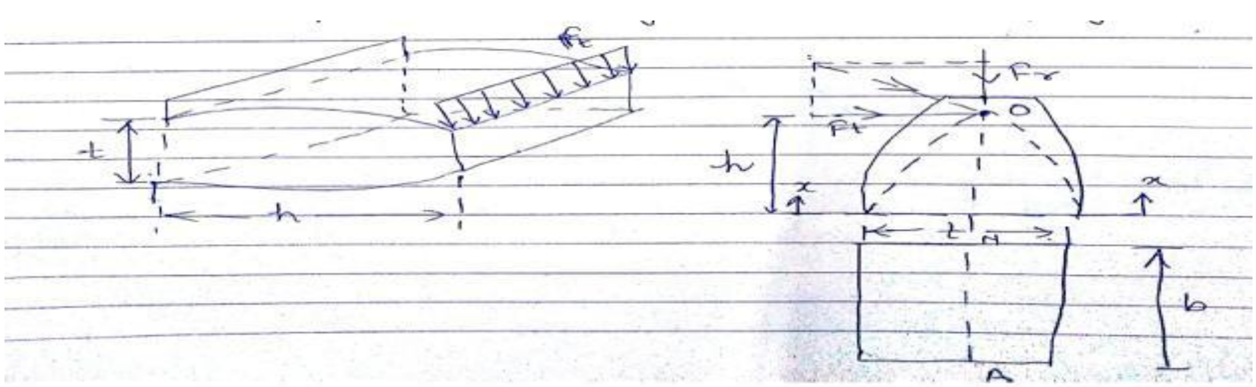
Assumption:

The lewis equ^n is based on the following assumption:

* The effect of the radial component (P_r) which induces compressive stress is neglected.
* It is assumed that the tangential component (P_t) is uniformly distributed over the face width of the gear.
* The effect of stress concentration is neglected.
* it is assumed that at any time only one pair of teeth is in contact and takes that total load.

* A parabola is contructed with in the tooth profile and shown dotted line. The advantage of parabolic outline is that it is a beam of uniform strength. For this beam the stress at any cross-section is uniform or same.

The weakest section of the gear tooth is at section XX, where the parabola is tangent to the tooth profile.



$\sigma_b = M_t \times u = F_t \times h \times t/.$

$\Rightarrow \quad M = F_t \times h \quad , \quad I = \frac{b t^3}{12}$

$y = t/2$

The bending stresses are given by,

$$\sigma_b = \frac{M \times y}{I} = \frac{F_t \times h \times t/2}{b \times t^3/12}$$

$$\sigma_b = \frac{F_t \times h \times 6}{b t^2} \qquad \boxed{F_t = \frac{\sigma_b \times b\, t^2}{6 h}}$$

In this expression, t and h are variables depending upon the size of the tooth (i.e the circular pitch) and its profile.

In this expression, t and h are variables depending upon the size of the tooth (i.e the circular pitch) and its profile.

Multiplying the numerator and denominator of the right hand side by P_c,

$$F_t = P_c \times b \times \sigma_b \times \frac{t^2}{6 \times h P_c}$$

$$\boxed{y = \frac{t^2}{6 h P_c}}$$

$$\boxed{F_t = \sigma_b \times b \times P_c \times y}$$

$$\boxed{F_t = \sigma_b \times b \times \pi m \times y}$$

or

$$\boxed{F_t = \sigma_b \times b \times m\, y}$$

Answer (2c)

Solution :

Given : $p = 45000$ W ; $n_p = 800$ r.p.m. ;

$G = 3.5$; $\phi = 20°$;

$z_p = 18$.

- **Assumptions and data selection :**

(i) The materials selected are :

For pinion : Plain carbon steel, 55C8 with $S_{up} = 720$ N/mm^2

For gear : Plain carbon steel, 45C8 with $S_{ug} = 630$ N/mm^2

(ii) For steel gear pair : C = 11500 e, N/mm

(iii) Assume, $N_f = 1.75$; $K_a = 1.5$; $K_m = 1.0$;

$b = 10\,m$; $K_v = \frac{6}{(6 + V)}$; IS grade = 6.

- **Beam strength :**

$$\sigma_{bp} = \frac{S_{up}}{3} = \frac{720}{3} = 240 \text{ N/mm}^2; \qquad \sigma_{bg} = \frac{S_{ug}}{3} = \frac{630}{3} = 210 \text{ N/mm}^2$$

$$G = \frac{n_p}{n_g} = \frac{z_g}{z_p} \quad ; \quad \therefore \quad 3.5 = \frac{z_g}{18}$$

$$\therefore \quad z_g = 63$$

$$Y_p = 0.484 - \frac{2.87}{z_p} = 0.484 - \frac{2.87}{18} = 0.3246$$

$$Y_g = 0.484 - \frac{2.87}{z_g} = 0.484 - \frac{2.87}{63} = 0.4384$$

$$\sigma_{bp} \cdot Y_p = 240 \times 0.3246 = 77.9 \text{ N/mm}^2$$

$$\sigma_{bg} \cdot Y_g = 210 \times 0.4384 = 92.07 \text{ N/mm}^2$$

As $\sigma_{bp} \cdot Y_p < \sigma_{bg} \cdot Y_g$, pinion is weaker than gear in bending. Hence it is necessary to design the pinion for bending.

$$F_b = \sigma_{bp} \cdot b \cdot m \cdot Y_p = 240 \times 10\,m \times m \times 0.3246$$

or $$F_b = 779.04\,m^2,\ N \qquad \text{...(a)}$$

Effective load :

$$d_p = m z_p = 18\,m$$

$$V = \frac{\pi d_p n_p}{60 \times 1000} = \frac{\pi \times 18\,m \times 800}{60 \times 1000} = 0.754\,m,\ m/s$$

$$F_t = \frac{P}{V} = \frac{45000}{0.754\,m} = \frac{59681.7}{m},\ N \qquad \text{...(b)}$$

$$K_v = \frac{6}{6+V} = \frac{6}{6+0.754\,m}$$

$$\therefore \quad F_{eff} = \frac{K_a \cdot K_m \cdot F_t}{K_v} = \frac{1.5 \times 1.0}{\left(\frac{6}{6+0.754\,m}\right)} \times \frac{59681.7}{m}$$

or $$F_{eff} = \frac{14920.42\,(6+0.754\,m)}{m},\ N \qquad \text{...(c)}$$

- **Estimation of module :**

In order to avoid the bending failure,

$$F_b = N_f \cdot F_{eff}$$

$$779.04\,m^2 = 1.75 \times \frac{14920.42\,(6+0.754\,m)}{m}$$

$$m^3 = 201.1 + 25.27\,m$$

Solving above equation by trial and error, we get,

$$m = 7.3 \text{ mm}$$

From Table 1.5.1, the standard value of module under first choice is 8 mm.

- **Dimensions of gear pair :**

$m = 8$ mm; $\quad z_p = 18$;

$z_g = 63$; $\quad b = 10\,m = 10 \times 8 = 80$ mm;

$d_p = m \cdot z_p = 8 \times 18 = 144$ mm; $\quad d_g = m \cdot z_g = 8 \times 63 = 504$ mm;

$a = (d_p + d_g)/2 = (144 + 504)/2 = 324$ mm;

$h_a = 1\,m = 8$ mm; $\quad h_f = 1.25\,m = 10$ mm. ...Ans.

- **Precise estimation of dynamic load by Buckingham's equation (check for design) :**

For grade 6, $e = 8.0 + 0.63\,[\,m + 0.25\sqrt{d}\,]$

For pinion, $e_p = 8.0 + 0.63\,[\,m + 0.25\sqrt{d_p}\,] = 8.0 + 0.63\,[\,8 + 0.25\sqrt{144}\,] = 14.93\ \mu m$

For gear, $e_g = 8.0 + 0.63\,[\,m + 0.25\sqrt{d_g}\,] = 8.0 + 0.63\,[\,8 + 0.25\sqrt{504}\,] = 16.576\ \mu m$

$e = e_p + e_g = 14.93 + 16.576 = 31.506\ \mu m = 31.506 \times 10^{-3}$ mm

The Buckingham's equation for the dynamic load in tangential direction is given by,

$$F_d = \frac{21\,V\,(\,bC + F_{t\,max}\,)}{21\,V + \sqrt{bC + F_{t\,max}}}$$

Now, $$F_t = \frac{59681.7}{m} = \frac{59681.7}{8} = 7460.2 \text{ N}$$

$$F_{t\,max} = K_a \cdot K_m \cdot F_t = 1.5 \times 1.0 \times 7460.2 = 11190.32 \text{ N}$$

$$V = 0.754\,m = 0.754 \times 8 = 6.032 \text{ m/s}$$

$$C = 11500\,e = 11500 \times 31.506 \times 10^{-3} = 362.32 \text{ N/mm}$$

$$\therefore \quad F_d = \frac{21 \times 6.032\,(\,80 \times 362.32 + 11190.32\,)}{21 \times 6.032 + \sqrt{80 \times 362.32 + 11190.32}}$$

or $$F_d = 15557.9 \text{ N}$$

- **Available factor of safety :**

$$F_{eff} = K_a \cdot K_m \cdot F_t + F_d = 1.5 \times 1.0 \times 7460.2 + 15557.9$$

or $$F_{eff} = 26748.2 \text{ N}$$

From Equation (a), $$F_b = 779.04\,m^2 = 779.04 \times (8)^2$$

or $$F_b = 49858.56 \text{ N}$$

Hence, the available factor of safety is,

$$N_f = \frac{F_w}{F_{eff}} = \frac{49858.56}{26748.2} \quad \text{or} \quad N_f = 1.86 > 1.75 \qquad \text{...An}$$

As the available factor of safety is higher than the required factor of safety, the design of safe.

Answer (2d) same steps used as question (2c)

SECTION – C

Answer (3a)

Involute	Cycloidal
Pressure angle remains same throughout the operation	Pressure angle keeps on changing during the operation. The angle is maximum at the start and end of engagement. It is zero at pitch point.
Teeth are weaker	Teeth are stronger
It is easier to manufacture due to convex surface	It is difficult to manufacture due to requirement of hypocycloid and epicycloids.
The velocity is not affected due to variation in centre distance	The centre distance should remains the same
Interference takes place	There is no interference
More wear and tear as contact takes place between convex surfaces	Less wear and tear as concave flank makes contact with convex flank.

Answer (3b)same steps used as question (2c)

Answer (4a)

Solution :

Given :

$\phi = 20°$; $G = 3$;

$n_p = 900$ r.p.m. ; $m = 3$ mm;

$b = 30$ mm ; $S_{ut} = 400$ N/mm^2;

$S_{yt} = 210$ N/mm^2 ; $N_f = 2.0$.

- **Assumptions and data selection :**

(i) Assume, $K_a = 1.0$; $K_m = 1.0$; $K_v = \frac{6}{(6 + V)}$.

(ii) For 20° full-depth involute, the minimum number of teeth on pinion is,

$$z_p = 18$$

- **Number of teeth on gear :**

$$G = \frac{z_g}{z_p}$$

$$3 = \frac{z_g}{18}$$

$$\therefore \quad z_g = 54$$

- **Beam strength :**

$$\sigma_{bp} = \frac{S_{up}}{3} = \frac{400}{3} = 133.33 \text{ N/mm}^2$$

$$Y_p = 0.484 - \frac{2.87}{z_p} = 0.484 - \frac{2.87}{18} = 0.3246$$

As pinion and gear are made of the same material, pinion is weaker than gear in bending. Her it is necessary to calculate the beam strength of pinion teeth.

$$F_b = \sigma_{bp} \cdot b \cdot m \cdot Y_p = 133.33 \times 30 \times 3 \times 0.3246$$

or $F_b = 3895.2$ N

$$F_b = 3895.2$$ ...A

- **Maximum static load that gear pair can transmit :**

$$V = \frac{\pi d_p n_p}{60 \times 1000} = \frac{\pi m z_p n_p}{60 \times 1000} = \frac{\pi \times 3 \times 18 \times 900}{60 \times 1000} = 2.545$$

$$K_v = \frac{6}{(6 + V)} = \frac{6}{(6 + 2.545)} = 0.7022$$

$$F_{eff} = \frac{K_a \cdot K_m \cdot F_t}{K_v} = \frac{1 \times 1 \times F_t}{0.7022}$$

or $F_{eff} = 1.424 F_t$

$$F_b = N_f \cdot F_{eff}$$

$$3895.2 = 2.0 \times 1.424 F_t$$

$$\therefore \quad F_t = 1367.69 \text{ N}$$ ...A

- **Rated power gear pair can transmit :**

$$P = F_t \cdot V = 1367.69 \times 2.545 = 3480.8 \text{ W}$$

or $P = 3.48$ kW

Answer (4b)

$$P = 10{,}000\ W \quad ; \quad n_p = 1440 \text{ r.p.m.} ;$$
$$K_a = 2 \quad ; \quad K_m = 1;$$
$$BHN = 400 \quad ; \quad N_f = 1.5.$$

- **Beam strength :**

$$\sigma_{bp} = \frac{S_{up}}{3} = \frac{600}{3} = 200\ N/mm^2$$
$$\sigma_{bg} = \frac{S_{ug}}{3} = \frac{400}{3} = 133.33\ N/mm^2$$

Assuming 20° full-depth involute tooth system,

$$Y_p = 0.484 - \frac{2.87}{z_p} = 0.484 - \frac{2.87}{20} = 0.3405$$

and
$$Y_g = 0.484 - \frac{2.87}{z_g} = 0.484 - \frac{2.87}{43} = 0.4172$$

Now, $\sigma_{bp} \cdot Y_p = 200 \times 0.3405 = 68.1\ N/mm^2$

and $\sigma_{bg} \cdot Y_g = 133.33 \times 0.4172 = 55.63\ N/mm^2$

As $\sigma_{bg} \cdot Y_g < \sigma_{bp} \cdot Y_g$, gear is weaker than pinion in bending. Hence it is necessary to design the gear for bending.

Let us assume $b = 10\ m$

$$\therefore \quad F_b = \sigma_{bg} \cdot b \cdot m \cdot Y_g = 133.33 \times 10\ m \times m \times 0.4172$$

or
$$F_b = 556.25\ m^2,\ N \qquad \text{...(a)}$$

- **Wear strength :**

$$d_p = m \cdot z_p = 20\ m$$
$$Q = \frac{2\ z_g}{z_g + z_p} = \frac{2 \times 43}{43 + 20} = 1.365$$

For steel pinion and steel gear,

$$K = 0.16\left[\frac{BHN}{100}\right]^2 = 0.16\left[\frac{400}{100}\right]^2 = 2.56\ N/mm^2$$
$$\therefore \quad F_w = d_p \cdot b \cdot Q \cdot K = 20\ m \times 10\ m \times 1.365 \times 2.56$$

or
$$F_w = 698.88\ m^2,\ N \qquad \text{...(b)}$$

or $F_w = 698.88\ m^2,\ N$

As $F_b < F_w$, gear pair is weaker in bending and hence it should be designed for the safety against bending failure.

- **Effective load :**

$$V = \frac{\pi d_p\ n_p}{60 \times 1000} = \frac{\pi \times 20\ m \times 1440}{60 \times 1000} = 1.508\ m/s$$
$$F_t = \frac{P}{V} = \frac{10000}{1.508\ m} = \frac{6631.3}{m},\ N$$

Assuming the gear pair is manufactured by hobbing, shaping or milling, the velocity factor can be taken as,

$$K_v = \frac{6}{6 + V} = \frac{6}{6 + 1.508\ m}$$
$$\therefore \quad F_{eff} = \frac{K_a \cdot K_m \cdot F_t}{K_v} = \frac{2 \times 1}{\left(\frac{6}{6 + 1.508\ m}\right)} \times \frac{6631.3}{m}$$

or
$$F_{eff} = \frac{2210.433\ (6 + 1.508\ m)}{m},\ N \qquad \text{...(c)}$$

- **Estimation of module :**

In order to avoid the bending failure,

$$F_b = N_f \cdot F_{eff} \qquad \text{...(d)}$$

Substituting Equations (a) and (c) in Equation (d),

$$556.25\ m^2 = 1.5 \times \frac{2210.433}{m}\ (6 + 1.508\ m)$$
$$m^3 = 35.764 + 8.989\ m$$

Solving above Equation by trial and error, we get,

$$m \approx 4.2\ mm$$

From Table 1.5.1, the standard value of module under first choice is 5 mm.

- **Dimensions of gear pair :**

$$\mathbf{m = 5\ mm \quad ; \quad z_p = 20 ;}$$
$$\mathbf{z_g = 43 \quad ; \quad b = 10\ m = 10 \times 5 = 50\ mm ;}$$
$$\mathbf{d_p = m \cdot z_p = 5 \times 20 = 100\ mm \quad ; \quad d_g = m \cdot z_g = 5 \times 43 = 215\ mm ;}$$
$$\mathbf{a = (d_p + d_g) / 2 = (100 + 215) / 2 = 157.5\ mm;}$$
$$\mathbf{h_a = 1\ m = 5\ mm \quad ; \quad h_f = 1.25\ m = 6.25\ mm.} \qquad \text{...Ans}$$

Answer (5a) same steps used as question (2c)

Answer (5b)

Causes of Gear tooth failure :-

The different modes of failure of gear teeth and their possible remedies to avoid the failure are as follows.

Bending failure :- Every gear tooth acts as a cantilever. if the total repetitive dynamic load acting on the gear tooth is greater than the beam strength of gear tooth, then the gear tooth will fail in bending, i.e. the gear tooth will break.

Avoid :- such failure, the module and face width of the gear is adjusted so that the beam strength is greater than the dynamic load.

Pitting :- it is the surface fatigue failure which occurs due to many repetition of Hertz contact stresses. The failure occurs when the surface contact stresses are higher than the Endurance limit the material.

Avoid :- the pitting, the dynamic load b/w the gear teeth should be less than the wear strength of the gear tooth.

Scoring :- The excessive heat is generated when there is an excessive surface pressure high speed or supply of lubricant fails. it is a stick slip phenomenon in which alternate shearing and welding takes place rapidly at high spots.

Avoid : it avoided by properly designing the parameter such as speed, pressure and proper flow of the lubricant, so that the temp at the rubbing faces is within the permissible limits.

Abrasive wear :- the foreign particles in the lubricants such as dirt, dust or burr enter b/w the tooth and damage the form of tooth.

Avoided - The lubricating oil or by using high viscosity lubricant oil which Enables the formation of thicker oil.

Corrosive wear :- The corrosion of the tooth surfaces is mainly caused due to the presence of corrosive elements such as additives present in the lubricating oils.

Avoid This type of wear proper anti - corrosive additives should be used.