Microwave Engineering (REC-601-solution of 1st sessional Examination 2019-20)

SECTION -A

Ans1a. Cut off frequency is defined as the lowest frequency below which wave does not propagate and EM wave propagation is possible at greater frequency than the cut off frequency.

Ans1b. communication system health is monitored by standing wave ratio (SWR) at transmitter side and signal to noise ratio (SNR) mostly at receiver side.

Ans 1c. Dominant mode in any wave guide has the lowest cut off frequency and Degenerate modes have the same cut off frequencies .

Ans1d. Waveguides are used as transmission media from transmitter to antenna and antenna to receiver in microwave frequency range.

Ans1e. Isolator does not allow the reflected waves from load side towards the source to prevent from damage of source.

SECTION-B

Ans2a

(a) **TE Modes in Rectangular wave guide :** the TE_{mn} modes in rectangular waveguide are characterized by $E_Z = 0$.and $H_Z \neq 0$ to have energy transmission in wave guide

$$H_z = H_{oz}\cos(m\pi x/a)\cos(n\pi y/b)e^{-j\beta z}$$

$$\nabla \times \boldsymbol{E} = \begin{bmatrix} \boldsymbol{a}_x & \boldsymbol{a_y} & \boldsymbol{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{bmatrix} = -\boldsymbol{j\omega\mu}(H_x\boldsymbol{a_x} + H_y\boldsymbol{a_y} + H_z\boldsymbol{a_z}) \text{-} \qquad 2$$

$$\nabla \times \boldsymbol{H} = \begin{bmatrix} \boldsymbol{a}_x & \boldsymbol{a_y} & \boldsymbol{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{bmatrix} = \boldsymbol{j\omega\varepsilon}(E_x\boldsymbol{a}_x + E_y\boldsymbol{a}_y + E_z\boldsymbol{a_z}) \text{-} \qquad 3$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \qquad 4$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\boldsymbol{j\omega\mu} H_y \qquad 5$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\boldsymbol{j\omega\mu} H_z \text{-} \qquad 6$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \boldsymbol{j\omega\varepsilon} E_x \qquad 7$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \boldsymbol{j\omega\varepsilon} E_y \qquad 8$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \boldsymbol{j\omega\varepsilon} E_z \qquad 9$$

We substitute $\frac{\partial}{\partial z} = -j\beta_g$ and $E_z = 0$ in the above equations and have as

$$\beta_g E_y = -\boldsymbol{\omega\mu} H_x \qquad 10$$

$$\beta_g E_x = \boldsymbol{\omega\mu} H_y \qquad 11$$

$$\frac{\partial H_z}{\partial y} + \boldsymbol{j\beta_g} H_y = \boldsymbol{j\omega\varepsilon} E_x \qquad 12$$

$$-\boldsymbol{j\beta_g} H_x - \frac{\partial H_z}{\partial x} = \boldsymbol{j\omega\varepsilon} E_y \qquad 13$$

Solving the above equations, we can have

$$E_x = \frac{\boldsymbol{-j\omega\mu}}{k_c^2} \frac{\partial H_z}{\partial y} \qquad 14$$

$$E_y = \frac{\boldsymbol{j\omega\mu}}{k_c^2} \frac{\partial H_z}{\partial x} \qquad 15$$

$$H_x = \frac{\boldsymbol{-j}\beta_g}{k_c^2} \frac{\partial H_z}{\partial x} \qquad 16$$

$$\boldsymbol{H_y} = \frac{\boldsymbol{-j}\beta_g}{k_c^2} \frac{\partial H_z}{\partial y} \qquad 17$$

where $k_c^2 = \omega^2 \mu\varepsilon - \beta_g^2$

Ans2b. **Losses in microstrip lines**:

The attenuation constant of the dominant microstrip mode depends on geometric factors, electrical properties of the substrate and conductors, and on the frequency. For a nonmagnetic dielectric substrate, two types of losses occur in the dominant microstrip mode: (1) dielectric loss in the substrate and (2) ohmic skin loss in the strip conductor and the ground plane. The sum of these two losses may be expressed as losses per unit length in terms of an attenuation factor α.

Dielectric losses:

when the conductivity of a dielectric cannot be neglected, the electric and magnetic fields in the dielectric are no longer in time phase. In that case the dielectric attenuation constant is expressed as

$$\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} \quad \text{Np/cm} \qquad 18$$

More appropriate formula is given by the following

$$\alpha_d = \left(\frac{4.34 q\sigma}{\sqrt{\varepsilon_{re}}}\right)\sqrt{\frac{\mu_0}{\varepsilon_0}} \quad \text{Np/cm} \qquad 19$$

where

$$q = \frac{\varepsilon_{re} - 1}{\varepsilon_r - 1} \qquad 20$$

q denotes the dielectric filling factor,

Ohmic losses:

In a microstrip line over a low-loss dielectric substrate, the predominant sources of losses at microwave frequencies are the non perfect conductors.

The microstrip conductor contributes the major part of the ohmic loss which can be expressed as

$$\alpha_c = \left(\frac{8.686\,Rs}{Zo\ w}\right) \quad \text{dB/cm} \quad \text{for } w < h \qquad 21$$

Radiation Loss:

In addition to the conductor and dielectric losses, microstrip line also has radiation losses. The radiation loss depends on the substrate's thickness and dielectric constant, as well as its geometry The ratio of radiated power to total dissipated power for an open-circuited micro strip line is expressed as

$$\frac{P\ rad}{Pt} = 240\pi^2\left(\frac{h}{\lambda}\right)^2 \frac{F(\varepsilon_{re})}{Z_0} \qquad 22$$

$$F(\varepsilon_{re}) = \frac{\varepsilon_{re}+1}{\varepsilon_{re}} - \frac{\varepsilon_{re}-1}{2\,\varepsilon_{re}\sqrt{\varepsilon_{re}}}\,ln\frac{\sqrt{\varepsilon_{re}}+1}{\sqrt{\varepsilon_{re}}-1} \qquad 23$$

The radiation factor decreases with increasing substrate dielectric constant.. The radiation loss decreases as the characteristic impedance increases.

The power delivered in the z direction by the guide is

$$P = \text{Re}\left[\frac{1}{2}\int_0^b\int_0^a (\mathbf{E}\times\mathbf{H}^*)\right]\cdot dx\,dy\,\mathbf{u}_z$$

$$= \frac{1}{2}\int_0^b\int_0^a\left[\left(E_{0y}\sin\left(\frac{\pi x}{a}\right)e^{-j\beta_g z}\,\mathbf{u}_y\right)\times\left(\frac{-\beta_g}{\omega\mu_0}E_{0y}\sin\left(\frac{\pi x}{a}\right)e^{+j\beta_g z}\mathbf{u}_x\right)\right]\cdot dx\,dy\,\mathbf{u}_z$$

$$= \frac{1}{2}E_{0y}^2\frac{\beta_g}{\omega\mu_0}\int_0^b\int_0^a\left(\sin\left(\frac{\pi x}{a}\right)\right)^2 dx\,dy$$

$$= \frac{1}{4}E_{0y}^2\frac{\beta_g}{\omega\mu_0}ab$$

$$373 = \frac{1}{4}E_{0y}^2\frac{193.5\pi(10^{-2})(2\times10^{-2})}{2\pi(3\times10^{10})(4\pi\times10^{7})}$$

$$E_{0y} = 53.87\ \text{kV/m}$$

The peak value of the electric intensity is 53.87 kV/m.

Ans2d. The Trigonometric equations for TMnp -mode are written as

$$H_r = H_{0r}J_n\left(\frac{X_{np}}{a}r\right)\sin(n\phi)e^{-j\beta_g z} \qquad 1$$

$$H_\phi = H_{0\phi}J_n'\left(\frac{X_{np}}{a}r\right)\cos(n\phi)e^{-j\beta_g z} \qquad 2$$

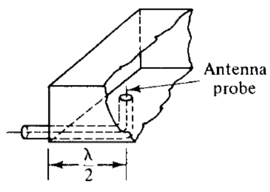
$$E_r = E_{0r}J_n'\left(\frac{X_{np}}{a}r\right)\cos(n\phi)e^{-j\beta_g z} \qquad 3$$

$$E_\phi = E_{0\phi}J_n\left(\frac{X_{np}}{a}r\right)\sin(n\phi)e^{-j\beta_g z} \qquad 4$$

$H_Z = 0$ 5

$E_z = E_{0z} J_n(k_c r) \cos(n\phi) e^{-j\beta_g z}$ 6

TE10 –mode is excited on to the broad surface by probe insertion vertically along y-axis at the position of maximum electric field which is most oftenly located at the centre of broader surface in XZ plane. It is shown in following figure.



TE_{10} mode

SECTION-C

Ans3a. TMnp –mode in circular waveguide: The TM_{np} modes in circular waveguide are characterized by $E_Z \neq 0$ and $H_Z = 0$ to have energy transmission in wave guide.

$E_z = E_{0z} J_n(k_c r) \cos(n\phi) e^{-j\beta_g z}$ 1

$J_n(k_c r)$ is an oscillating function having infinite roots.

By applying the boundary condition; at r=a,the electric field component in z-direction E_z becomes zero

$J_n(k_c a) = 0$ 2

$k_c a = X_{np}$ 3

The root of this equation is denoted by

$X_{np} H_z = H_{0z} J_n' \left(\frac{X_{np}}{r} a\right) \cos(n\phi) e^{-j\beta_g z}$ 4

Table 1.1 pth roots of $J_n(k_c a) = 0$ for TMnp -modes

p	n=0	1	2	3
1	2.405	3.832	5.136	6.380
2	5.520	7.106	8.417	9.761
3	8.645	10.173	11.620	13.015
4	11.792	13.32	14.796	

$$\nabla \times \boldsymbol{E} = \frac{\boldsymbol{1}}{\boldsymbol{r}}\begin{bmatrix} \boldsymbol{a}_r & r\boldsymbol{a}_{\boldsymbol{\phi}} & \boldsymbol{a}_{\boldsymbol{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ E_r & rE_\phi & E_z \end{bmatrix} = -\boldsymbol{j\omega\mu}\left(H_r \boldsymbol{a}_{\boldsymbol{r}} + H_\phi \boldsymbol{a}_{\boldsymbol{\phi}} + H_z \boldsymbol{a}_{\boldsymbol{z}}\right) \quad 5$$

$$\nabla \times \boldsymbol{H} = \frac{\boldsymbol{1}}{\boldsymbol{r}}\begin{bmatrix} \boldsymbol{a}_r & r\boldsymbol{a}_{\boldsymbol{\phi}} & \boldsymbol{a}_{\boldsymbol{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ H_r & rH_\phi & H_z \end{bmatrix} = \boldsymbol{j\omega\varepsilon}\left(E_r \boldsymbol{a}_r + E_\phi \boldsymbol{a}_\phi + E_z \boldsymbol{a}_{\boldsymbol{z}}\right) \quad 6$$

$$\frac{\boldsymbol{1}}{\boldsymbol{r}} \frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} = -\boldsymbol{j\omega\mu} H_r \quad 7$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -\boldsymbol{j\omega\mu} H_\phi \quad 8$$

$$\frac{\boldsymbol{1}}{\boldsymbol{r}} \frac{\partial (rE_\phi)}{\partial r} - \frac{\boldsymbol{1}}{\boldsymbol{r}} \frac{\partial E_r}{\partial \phi} = -\boldsymbol{j\omega\mu} H_z \quad 9$$

$$\frac{\boldsymbol{1}}{\boldsymbol{r}} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} = \boldsymbol{j\omega\varepsilon} E_r \quad 10$$

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = \boldsymbol{j\omega\varepsilon} E_\phi \quad 11$$

$$\frac{\boldsymbol{1}}{\boldsymbol{r}} \frac{\partial (rH_\phi)}{\partial r} - \frac{\boldsymbol{1}}{\boldsymbol{r}} \frac{\partial H_r}{\partial \phi} = \boldsymbol{j\omega\varepsilon} E_z \quad 12$$

We substitute $\frac{\partial}{\partial z} = -j\beta_g$ and $H_z = 0$ in the above equations and have as

$\beta_g H_\phi = \boldsymbol{\omega\varepsilon} E_r$ 137

$$\beta_g H_r = -\boldsymbol{\omega\varepsilon} E_\phi \quad 13$$

$$\frac{\boldsymbol{1}}{\boldsymbol{r}} \frac{\partial E_z}{\partial \phi} + \boldsymbol{j\beta_g} E_\phi = -\boldsymbol{j\omega}\square H_r \quad 14$$

$$\boldsymbol{j\beta_g} E_r + \frac{\partial E_z}{\partial r} = \boldsymbol{j\omega}\square H_\phi \quad 15$$

Solving the above equations ,we can have

$$H_r = \frac{\boldsymbol{j\omega\varepsilon}}{\boldsymbol{r}k_c^2} \frac{\partial E_z}{\partial \phi} \quad 16$$

$$H_\phi = \frac{-\boldsymbol{j\omega\varepsilon}}{k_c^2} \frac{\partial E_z}{\partial r} \quad 17$$

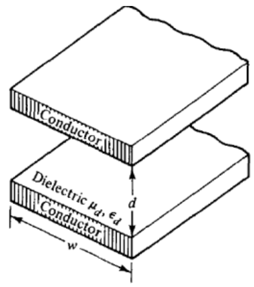
$$H_Z = 0$$

$$E_r = \frac{-\boldsymbol{j}\beta_g}{k_c^2} \frac{\partial E_z}{\partial r} \quad 18$$

$E_\phi = \frac{-\boldsymbol{j}\beta_g}{\boldsymbol{r}k_c^2}\frac{\partial E_z}{\partial \phi}$

Ans3b ***PARALLEL STRIP LINES***

A parallel strip line consists of two perfectly parallel strips separated by a perfect dielectric slab of uniform thickness, as shown in the following figure. The plate width is *w*, the separation distance is *d*, and the relative dielectric constant of the slab is $\boldsymbol{\varepsilon_{rd}}$.



Distributed Parameters

In a microwave integrated circuit a strip line can be easily fabricated on a dielectric substrate by using printed-circuit techniques. A parallel stripline is similar to a two-conductor transmission line, so it can support a quasi-TEM mode. Consider a TEM-mode wave propagating in the positive *z* direction in a lossless strip line (*R* = G = O). The electric field is in the y direction, and the magnetic field is in the *x* direction. If the width *w* is much larger than the separation distance d, the fringing capacitance is negligible. Thus the equation for the inductance along the two conducting strips can be written as

$$L = \frac{\mu\, d}{w} \text{ H/m}$$

where μ is the permeability of the conductor. The capacitance between the two conducting strips can be expressed as

$$C = \frac{\varepsilon\, w}{d} \text{ F/m}$$

)

where ε is the permittivity of the dielectric slab.If the two parallel strips have some surface resistance and the dielectric substrate has some shunt conductance, however, the parallel stripline would have some losses. The series resistance for both strips is given by

$$R = \frac{2\,Rs}{w} = \frac{2}{w}\sqrt{\frac{?\pi f}{\sigma}}$$

where *Rs* is the conductor surface resistance . where σ is the conductivity of the dielectric substrate.

$$R = \frac{w \quad \sigma}{d}$$

Characteristic Impedance

The characteristic impedance of a lossless parallel strip line is

$$Z = \sqrt{\frac{L}{C}}$$

Attenuation Losses

The propagation constant of a parallel strip line at microwave frequencies can be expressed By

$$\Upsilon = \sqrt{(R + jwL)(G + jwC)}$$

for $R \ll wL$ and $G \ll wC$

Thus the attenuation and phase constants are

$$\alpha = R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}} \quad \text{Np/m and}$$

$$\beta = \omega\sqrt{LC} \text{ rad/m}$$

Ans 4a. TE mn –mode Impedance is expressed as

$$Z_g = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega\mu}{\beta_g} = \frac{\eta}{\sqrt{1 - (f_c/f)^2}}$$

TM mn –mode Impedance equation is expressed as

$$Z_g = \frac{\beta_g}{\omega\epsilon} = \eta\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

Cut off frequency is given as

$$f_c = \frac{X'_{np}}{2\pi a\sqrt{\mu\epsilon}} = 1.841 \text{ x } 3\text{x } 10^8/ 2\text{x}3.14\text{x}2.93\text{x}10^{-2}=3\text{GHz}$$

Thus $\lambda_c = 3\text{x}10^8/f_c = 3\text{x}10^8/3\text{x}10^9 = 0.1\text{m} = 10\text{cm}$

Ans4b. E-plane tee (series tee).

An E-plane tee is a waveguide tee in which the axis of its side arm is parallel to the *E* field of the main guide (see Fig.2.3). If the collinear arms are symmetric about the side arm, there are two different transmission characteristics (see Fig2.3(a) &(b). It can be seen that if the *E* - plane tee is perfectly matched with the aid of screw tuners or inductive or capacitive windows at the junction, the diagonal components of the scattering matrix, S11, S22, and S33, are zero because there will be no reflection. When the waves are fed into the side arm (port 3), the waves appearing at port l and port 2 of the collinear arm will be in opposite phase and in the same magnitude.

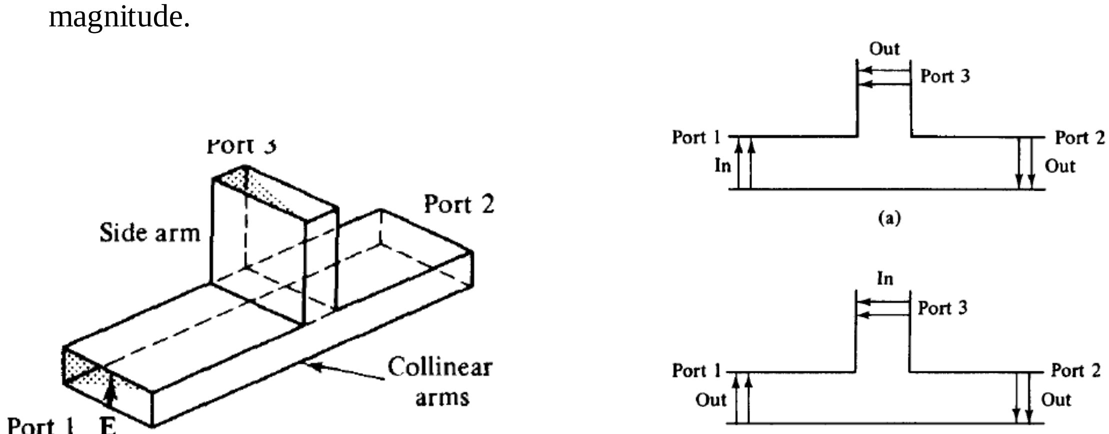


Figure 2.3

S –Matrix for E-Plane TEE;

S-matrix for E-plane Tee is 3x3 as there are 3-ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad 1$$

The terms from S11 to S33 are called transmission coefficients. By applying the properties of S-matrix , we have to calculate the values of these coefficients.

(i) From symmetry property,

$S_{ij} = S_{ji}$ 2

$S_{11} = S_{21}$ 3

$S_{13} = S_{31}$ 4

$S_{23} = S_{32}$ 5

(ii) if port 3 is perfectly matched to the junction

$S_{13} = 0$ 6

(iii) for an input at port 3 ,the outputs at port 1 and port 2 are out of phase by 180 degree.

$S_{23} = -S_{31},$ 7

Using the above properties, we have

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \quad 8$$

Using the unitary properties

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 9$$

$S_{11}S_{11}^* + S_{12}S_{12}^* + S_{13}S_{13}^* = 1$ 10

OR

$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$ 11

$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$ 12

$|S_{13}|^2 + |S_{13}|^2 = 1$ 13

From equation(19)

$S_{13} = \frac{1}{\sqrt{2}}$ 14

From equations 11 and 12,we get

$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2$ 15

$S_{11} = S_{22}$ 16

Multiplying column third of $[S]$ and row of matrix $[S]^*$ and using zero property of $[S]$ Matrix, we get

$$S_{13}S_{11}^* - S_{13}S_{12}^* = 0 \qquad 17$$

$$S_{13}(S_{11}^* - S_{12}^*) = 0 \qquad 18$$

$$S_{11} = S_{22} = S_{12} \qquad 19$$

$$|S_{11}|^2 + |S_{11}|^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1 \qquad 20$$

$$S_{11} = \frac{1}{2} \qquad 21$$

Substituting all these values in matrix 8

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \qquad 22$$

Ans5a For rectangular wave guide , the resonant frequency is expressed as

$$f_r = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\}}$$

m=1,n=0,p=1,a=2cm,b=1cmand d=3cm

so fr = 9GHz

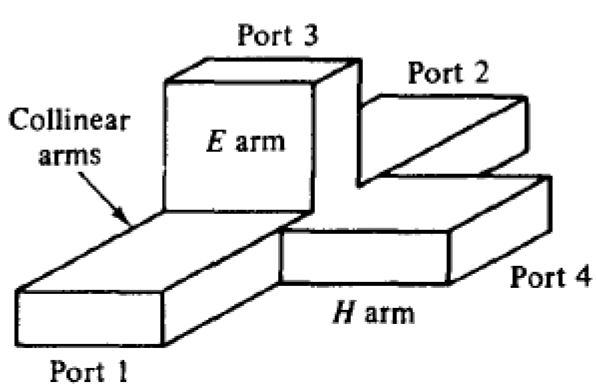
For circular wave guide in TM mode , the resonant frequency is expressed as

$$f_r = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\{\left(\frac{X_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2\}}$$

For m=0,n=1,d=2

So,fr= 6.27GHz.

Ans5b. Magic Tee is shown in following figure .



S-matrix for magic Tee is given as

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \qquad 1$$

(i)Due to Symmetry property of S- matrix, we have

$S_{12} = S_{21},\ S_{13} = S_{31},\ S_{14} = S_{41},\ S_{23} = S_{32},\ S_{24} = S_{42},\ S_{34} = S_{43}$ 2

(ii) Due to E-plane Tee section,

$S_{13} = -S_{23}$ 3

(iii) Due to H-plane Tee

$S_{24} = S_{14}$ 4

(iv) because of the geometry of the junction ,an input at port4 can not come out of Port 4 (as they are isolated) and vice -versa

$S_{34} = S_{43} = 0$ 5

(v) if the ports 3 and 4 are perfectly matched to the junction

$S_{33} = S_{44} = 0$ 6

Substituting the above properties in matrix (2)

$$[S][S]^* = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & -S_{13} & S_{14} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* & S_{14}^* \\ S_{13}^* & -S_{13}^* & 0 & 0 \\ S_{14}^* & S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad 7$$

$S_{11}S_{11}^* + S_{12}\,S_{12}^* + S_{13}S_{13}^* + S_{14}S_{14}^* = 1$ 8

OR

$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$ 9

$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$ 10

$|S_{13}|^2 + |S_{13}|^2 = 1$ 11

$|S_{14}|^2 + |S_{14}|^2 = 1$ 12

$S_{13} = \frac{1}{\sqrt{2}} = S_{14}$ 13

$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2$ 14

$S_{11} = S_{22}$ 15

$$|S_{11}|^2 + |S_{12}|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 = 1 \quad 16$$

$$|S_{11}|^2 + |S_{12}|^2 = 0 \quad 17$$

$$S_{11} = S_{12} = 0 \quad 18$$

From equation (15) , $S_{22} = 0$ substitution the values of S-parameters in (1)

$$[S] = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$