Subject Code : KAS-402 **Subject :Engg. Math-IV**

B.Tech. - SEMESTER 4

Branch : CSE and EE

Time –1hr 30 mins **Maximum Marks – 30**

SECTION – A

1. Attempt all questions in brief. (1*5 = 5)

Q N	solutions
a.	Write the formula of measure of Kurtosis. **Soln**. defines degree of peakness. It is of 3 types.
b.	Discuss the types of Correlation **Soln** .Define positive and negative correlations
c.	Write the normal equations of $y = ax^2 + \frac{b}{x}$. **Soln.** $\boldsymbol{\sum x^2y = \sum ax^4 + \sum bx, \sum y/x = \sum ax + \sum b/x^2}$
d.	Find the probability of throwing 10 with 2 dice. **Soln. 1/12**
e.	State Baye's Theorem. *If $E_1, E_2, \ldots\ldots, E_n$ are mutually exclusive and exhaustive events with $P(E_i) \neq 0$, $(i = 1, 2, \ldots\ldots, n$ of a random experiment then for any arbitrary event A of the sample space of the above experiment with $P(A) > 0$, we have* $P(E_i/A) = \dfrac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}$.

SECTION - B

2. Attempt any <u>TWO</u> of the following. (2*5 = 10)

Q N	solutions
a.	In a certain distribution, the first 4 moments about the point x=4 are -1.5, 17, -30, and 308. Find the moments about mean and about origin. Also, calculate β_1 *and* β_2.

Sol. We have, $A = 4, \mu'_1 = -1.5, \mu'_2 = 17, \mu'_3 = -30, \mu'_4 = 308$

Moments about mean

$$\mu_1 = 0$$

$$\mu_2 = \mu'_2 - \mu_1'^2 = 17 - (-1.5)^2 = 14.75$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3 = -30 - 3(17)(-1.5) + 2(-1.5)^3 = 39$$

$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu_1'^2 - 3\mu_1'^4$$

$$= 308 - 4(-30)(-1.5) + 6(17)(-1.5)^2 - 3(-1.5)^4 = 342.312$$

Moments about origin

$$\nu_1 = \bar{x} = \mu'_1 + A = -1.5 + 4 = 2.5$$

$$\nu_2 = \mu_2 + \bar{x}^2 = 14.75 + (2.5)^2 = 21$$

$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3 = 166$$

$$\nu_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4 = 1332$$

alculation of β_1 and β_2

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0.492377 \qquad \beta_2 = \frac{\mu_4}{\mu_2^2} = 1.573398$$

b.

4 cards are drawn from a pack of cards. Find the probability that (i) all are diamonds, (ii) there is one card of each suit, and (iii) there are 2 spades and two hearts.

Sol. 4 cards can be drawn from a pack of 52 cards in ${}^{52}C_4$ ways.

$\therefore$ Exhaustive number of cases $= {}^{52}C_4 = \frac{52 \times 51 \times 50 \times 49}{4 \times 3 \times 2 \times 1} = 270725.$

(*i*) There are 13 diamonds in the pack and 4 can be drawn out of them in ${}^{13}C_4$ ways.

$\therefore$ Favourable number of cases $= {}^{13}C_4 = \frac{13 \times 12 \times 11 \times 10}{4 \times 3 \times 2 \times 1} = 715.$

Required probability $= \frac{715}{270725} = \frac{143}{54145} = \frac{11}{4165}$.

(*ii*) There are 4 suits, each containing 13 cards.

$\therefore$ Favourable number of cases $= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = 13 \times 13 \times 13 \times 13.$

Required probability $= \frac{13 \times 13 \times 13 \times 13}{270725} = \frac{2197}{20825}$.

(*iii*) 2 spades out of 13 can be drawn in ${}^{13}C_2$ ways.

2 hearts out of 13 can be drawn in ${}^{13}C_2$ ways.

$\therefore$ Favourable number of cases $= {}^{13}C_2 \times {}^{13}C_2 = 78 \times 78$

Required probability $= \frac{78 \times 78}{270725} = \frac{468}{20825}$.

c.

The pressure of the gas corresponding to various volumes V is measured, given by the following data. Fit the data to the equation $PV^{\gamma} = C$

V(cm^3)	50	60	70	90	100
P(kgcm^{-2}	64.7	51.3	40.5	25.5	78

Fit the data to the

Sol.

$$PV^{\gamma} = C$$

$\Rightarrow$

$$P = CV^{-\gamma}$$

Taking log on both sides, we get

$$\log P = \log C - \gamma \log V$$

$\Rightarrow$

$$Y = A + BX$$

where, $Y = \log P$, $A = \log C$, $B = -\gamma$, $X = \log V$

Normal equations are

$$\Sigma Y = mA + B\Sigma X$$

and

$$\Sigma XY = A\Sigma X + B\Sigma X^2$$

Here $m = 5$

The table is as below:

V	P	$X = \log V$	$Y = \log P$	XY	X^2
50	64.7	1.69897	1.81090	3.07666	2.88650
60	51.3	1.77815	1.71012	3.04085	3.16182
70	40.5	1.84510	1.60746	2.96592	3.40439
90	25.9	1.95424	1.41330	2.76193	3.81905
100	78.0	2	1.89209	3.78418	4
		$\Sigma X = 9.27646$	$\Sigma Y = 8.43387$	$\Sigma XY = 15.62954$	$\Sigma X^2 = 17.27176$

d.

If there are 3 containers B_1, B_2, B_3 having 4 red, 3 white and 2 blue balls; 1 red, 2 white and 3 blue balls; 3 red, 4 white and 1 blue balls. If one ball is taken out from a randomly chosen container, the colour of the ball is found to be red. What is the probability that the ball was from B_2 container?

the ball was from B_2 container?

Solution. Here B_i represents i^{th} container chosen and R represent event of taking out the Red ball, therefore

$$P(B_1) = P(B_2) = P(B_3) = 1/3$$

While

$$P(R \mid B_1) = 4/9, \; P(R \mid B_2) = 1/6$$

$$P(R \mid B_3) = 3/8$$

Hence by Bayes' theorem, we get

$$P(B_2|R) = \frac{P(B_2).P(R|B_2)}{\sum_{i=1}^{3} P(B_i).P(R|B_i)}$$

$$= \frac{\left(\frac{1}{3}\right).\left(\frac{1}{6}\right)}{\left(\frac{1}{3}\right)\left(\frac{4}{9}\right)+\left(\frac{1}{3}\right).\left(\frac{1}{6}\right)+\left(\frac{1}{3}\right).\left(\frac{3}{8}\right)}$$

SECTION - C

3. Attempt any ONE part of the following : (1*5 = 5)

<table>
<tr><th>Q N</th><th>solutions</th></tr>
<tr><td>a.</td><td>The two regression equations of the variables are $x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$ Find (i) the mean of x and y (ii) the correlation coefficient between x and y.

Solution. $x = 19.13 - 0.87y$
$y = 11.64 - 0.50x$
As (1) and (2) pass through $(\bar{x}, \bar{y})$:
$\bar{x} = 19.13 - 0.87\bar{y}$
$\bar{y} = 11.64 - 0.50\bar{x}$
On solving (3) and (4) we get
$\bar{x} = 15.937,\ \bar{y} = 3.67$
From (1) $r\dfrac{\sigma_x}{\sigma_y} = -0.87$
From (2) $r\dfrac{\sigma_y}{\sigma_x} = -0.50$
As σ_x and σ_y are always positive, so r is negative.
Multiplying (5) and (6), we get
$r\dfrac{\sigma_x}{\sigma_y} \cdot r\dfrac{\sigma_y}{\sigma_x} = -0.87 \times (-0.50)$
$r^2 = 0.435 \quad \Rightarrow \quad r = -0.66$</td></tr>
<tr><td>b.</td><td>Calculate the coefficient of skewness from the following data:
<table>
<tr><td>Wages in Rs.</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td><td>50-60</td><td>60-70</td></tr>
<tr><td>No. of labours</td><td>185</td><td>77</td><td>34</td><td>180</td><td>136</td><td>23</td><td>50</td></tr>
</table></td></tr>
</table>

Class	f_i	Mid value x_i	$x_i - 35$	$f_i(x_i - 35)$	$f_i(x_i - 35)^2$	$f_i(x_i - 35)^3$
0-10	185	5	– 30	– 5550	– 166500	– 4995000
10-20	77	15	– 20	– 1540	– 30800	– 616000
20-30	34	25	– 10	– 340	3400	– 34000
30-40	180	35	0	0	0	0
40-50	136	45	10	1360	13600	136000
50-60	23	55	20	460	9200	184000
60-70	50	65	30	1500	45000	1350000
	$\Sigma f_i = 685$			$\Sigma f_i(x_i - 35) = -4110$	$\Sigma f_i(x_i - 35)^2 = 268500$	$\Sigma f_i(x_i - 35)^3 = -3975000$

$$\mu_1' = \frac{\Sigma f(x-35)}{\Sigma f_i} = \frac{-4110}{685} = -6$$

$$\mu_2' = \frac{\Sigma f(x-35)^2}{\Sigma f_i} = \frac{268500}{685} = 391.97$$

$$\mu_3' = \frac{\Sigma f(x-35)^3}{\Sigma f_i} = \frac{-3975000}{685} = -5802.9$$

$$\bar{x} = a + \mu_1' = 35 - 6 = 29$$

$$\mu_2 = \mu_2 - \mu_1'^2 = 391.97 - 36 = 355.97$$

$$\begin{aligned}\mu_3 &= \mu_3' - 3\mu_1' \mu_1' + 2\mu_1'^3 \\ &= -5802.9 - 3 \times 391.97 \times (-6) + 2(-6)^3 \\ &= -5802.9 + 7055.46 - 432 \\ &= -6234.9 + 7055.46 \\ &= 820.56\end{aligned}$$

$$\gamma_1 = \sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{820.56}{\sqrt{(355.97)^3}} = \frac{820.56}{\sqrt{45106610.72}}$$

$$= \frac{820.56}{6716.15} = 0.1223$$

Ans.

4. Attempt any ONE part of the following : (1*5 = 5)

QN	
a.	Three machines I, II, and III manufacture respectively 0.4, 0.5 and 0.1 of the total production. The percentage of defectives items produced by machines I, II and III is 2, 4 and 1 percent respectively. For an item chosen at random, what is the probability it is defective?

cent respectively. For an item chosen at random, ...

Solution. The defective item produced by machine I $= \dfrac{0.4\times 2}{100} = \dfrac{0.8}{100}$

The defective item produced by machine II $= \dfrac{0.5\times 4}{100} = \dfrac{2}{100}$.

The defective item produced by machine III $= \dfrac{0.1\times 1}{100} = \dfrac{0.1}{100}$

The required probability $= \dfrac{0.8}{100} + \dfrac{2}{100} + \dfrac{0.1}{100} = \dfrac{2.9}{100} = 0.029$

---

b.

A can hit a target 4 times in 5 shots; B 3 shots; C twice in 3 shots. They fire a volley. What is the probability that at least 2 shots hit?

(*iv*) B, C hit the target and A misses it, the probability for which is

$$\left(1-\frac{4}{5}\right)\times\frac{3}{4}\times\frac{2}{3} = \frac{1}{5}\times\frac{3}{4}\times\frac{2}{3} = \frac{6}{60}.$$

Since these are mutually exclusive events,

uired probability $= \dfrac{24}{60} + \dfrac{12}{60} + \dfrac{8}{60} + \dfrac{6}{60} = \dfrac{50}{60} = \dfrac{5}{6}.$

Sol. Probability of A's hitting the target $= \dfrac{4}{5}$

Probability of B's hitting the target $= \dfrac{3}{4}$

Probability of C's hitting the target $= \dfrac{2}{3}$.

For at least two hits, we may have

(*i*) A, B, C all hit the target, the probability for which is $\dfrac{4}{5}\times\dfrac{3}{4}\times\dfrac{2}{3} = \dfrac{24}{60}$.

(*ii*) A, B hit the target and C misses it, the probability for which is

$$\frac{4}{5}\times\frac{3}{4}\times\left(1-\frac{2}{3}\right) = \frac{4}{5}\times\frac{3}{4}\times\frac{1}{3} = \frac{12}{60}.$$

(*iii*) A, C hit the target and B misses it, the probability for which is

$$\frac{4}{5}\times\left(1-\frac{3}{4}\right)\times\frac{2}{3} = \frac{4}{5}\times\frac{1}{4}\times\frac{2}{3} = \frac{8}{60}.$$

5. Attempt any ONE part of the following : (1*5 = 5)

Q N	solutions

a. Fit a second degree parabola to the following:

x	1	2	3	4	5
y	1090	1220	1390	1625	1915

(R.G.P.V., Bhopal, III Semester, Dec. 2003)

Solution. Let the equation of the parabola be

$$y = a + bx + cx^2 \quad \text{...(1)}$$

x	y	xy	x^2	x^2y	x^3	x^4
1	1090	1090	1	1090	1	1
2	1220	2440	4	4880	8	16
3	1390	4170	9	12510	27	81
4	1625	6500	16	26000	64	256
5	1915	9575	25	47875	125	625
$\Sigma x = 15$	$\Sigma y = 7240$	$\Sigma xy = 23775$	$\Sigma x^2 = 55$	$\Sigma x^2y = 92355$	$\Sigma x^3 = 225$	$\Sigma x^4 = 979$

Normal equations are

$$\Sigma y = na + b\Sigma x + c\Sigma x^2 \quad \text{...(2}$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{...(3}$$
$$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad \text{...(4}$$

On putting the values of $n, \Sigma x, \Sigma x^2, \Sigma x^3, \Sigma x^4, \Sigma y, \Sigma xy, \Sigma x^2y$, in (3), (4) and (5), we get

$$7240 = 5a + 15b + 55c \quad \text{...(5}$$
$$23775 = 15a + 55b + 225c \quad [\because n = 5] \quad \text{...(}$$
$$92355 = 55a + 225b + 979c \quad \text{...(}$$

Steps for solution of (5), (6) and (7) are the following:

$3 \times (5)$, $\quad 21720 = 15a + 45b + 165c \quad$ ...(

$(6) - (8)$, $\quad 2055 = 10b + 60c \quad$ ...

$11 \times (5)$, $\quad 79640 = 55a + 165b + 605c \quad$ ...(

$(7) - (10)$, $\quad 12715 = 60b + 374c \quad$ ...(

$6 \times (9)$, $\quad 12330 = 60b + 360c \quad$ ...(

$(11) - (12)$, $\quad 385 = 14c \quad \Rightarrow \quad c = \frac{55}{2}$

From (9), $\quad 2055 = 10b + 60\left(\frac{55}{2}\right) \quad \Rightarrow \quad b = \frac{81}{2}$

From (5), $\quad 7240 = 5a + 15\left(\frac{81}{2}\right) + 55\left(\frac{55}{2}\right) \quad \Rightarrow \quad a = 1024$

On putting the values of a, b, c in (1), we get

$$y = 1024 + \frac{81}{2}x + \frac{55}{2}x^2$$

The equation of the required parabola is

$$2y = 2048 + 81x + 55x^2$$

b. Obtain the rank correlation coefficient for the following data.

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

X	68	64	75	50	64	80	75	40	55	64	Total
Y	62	58	68	45	81	60	68	48	50	70	
Ranks in X (x)	4	6	2.5	9	6	1	2.5	10	8	6	
Ranks in Y (y)	5	7	3.5	10	1	6	3.5	9	8	2	
$D = x - y$	−1	−1	−1	−1	5	−5	−1	1	0	4	0
D^2	1	1	1	1	25	25	1	1	0	16	72

In the X-series, the value 75 occurs twice. Had these values been slightly different, they would have been given the ranks 2 and 3. Therefore, the common rank given to them is $\frac{2+3}{2}$ = 2.5. The value 64 occurs thrice. Had these values been slightly different, they would have been given the ranks 5, 6, and 7. Therefore the common rank given to them is $\frac{5+6+7}{3} = 6$ Similarly, in the Y-series, the value 68 occurs twice. Had these values been slightly different they would have been given the ranks 3 and 4? Therefore, the common rank given to them i $\frac{3+4}{2} = 3.5$.

Thus, m has the values 2, 3, 2.

$$\therefore \qquad r = 1 - \frac{6\left\{\Sigma D^2 + \frac{1}{12} m_1(m_1^2 - 1) + \frac{1}{12} m_2(m_2^2 - 1) + \frac{1}{12} m_3(m_3^2 - 1)\right\}}{n(n^2 - 1)}$$

$$= 1 - \frac{6\left[72 + \frac{1}{12}.2(2^2 - 1) + \frac{1}{12}.3(3^2 - 1) + \frac{1}{12}.2(2^2 - 1)\right]}{10(10^2 - 1)}$$

$$= 1 - \left\{\frac{6 \times 75}{990}\right\} = \frac{6}{11} = 0.545.$$